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DoQSS Working Paper No. 12-05
June 2012

Leading education and social research
Institute of Education
University of London

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Reducing bias due to missing values of the response variable by joint modeling with an auxiliary variable

Alfonso Miranda[*], **Sophia Rabe-Hesketh**[†], **John W. McDonald**[‡§]

Abstract. In this paper, we consider the problem of missing values of a continuous response variable that cannot be assumed to be missing at random. The example considered here is an analysis of pupil's subjective engagement at school using longitudinal survey data, where the engagement score from wave 3 of the survey is missing due to a combination of attrition and item non-response. If less engaged students are more likely to drop out and less likely to respond to questions regarding their engagement, then missingness is not ignorable and can lead to inconsistent estimates. We suggest alleviating this problem by modelling the response variable jointly with an auxiliary variable that is correlated with the response variable and not subject to non-response. Such auxiliary variables can be found in administrative data, in our example, the National Pupil Database containing test scores from national achievement tests. We estimate a joint model for engagement and achievement to reduce the bias due to missing values of engagement. A Monte Carlo study is performed to compare our proposed multivariate response approach with alternative approaches such as the Heckman selection model and inverse probability of selection weighting.

JEL classification: C13, C33, I21.

Keywords: Auxiliary variable, joint model, multivariate regression, not missing at random, sample selection bias, seemingly-unrelated regressions, selection model, SUR.

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[§]This research was supported by ESRC grant RES-576-25-0014 by the ESRC National Centre for Research Methods ADMIN node at the Institute of Education.

1. Introduction

Olsen (2006) pointed out that "Perhaps the greatest unexploited opportunity for survey projects lies in administrative data." In this paper, we analyse an incomplete continuous survey response when the survey data are linked to complete administrative data. Linkage between survey and administrative data opens possibilities for new strategies to handle missing survey data. When missingness cannot be assumed to be ignorable, one strategy follows the recommendations by Little and Rubin (1999, p. 1130) who stated that

> "Given the problems inherent in nonignorable modeling, we have generally advocated trying to make the ignorability assumption as plausible as possible by collecting as much information about incomplete cases as possible, and then including this information for inferences via model-based approaches such as multiple imputation."

This strategy is usually interpreted as including the auxiliary information as extra control or explanatory variables to make a missing at random (MAR) assumption more realistic. Instead, we exploit the linked data by choosing a continuous variable recorded in the administrative data that is strongly correlated with the continuous survey response. We propose using this auxiliary administrative variable as an additional response and modelling it jointly with the survey response. The model is a multivariate regression or seemingly unrelated regresion (SUR) model in which the error terms of the response and auxiliary variable are allowed to be correlated. The parameters are estimated by maximum likelihood, exploiting all the data available.

To the best of our knowledge, our proposed strategy for handling missing survey data has not been discussed before. This approach is attractive when, besides the survey, researchers have access to high quality supplementary information such as administrative data. Administrative data are usually well maintained and often complete, i.e., do not suffer from item non-response and/or attrition. Our proposed joint modelling strategy may be used with either cross-sectional or longitudinal surveys with missing data. The approach is particularly attractive given the recent increasing availability of linked survey

and administrative data for scientific research. For example, in the USA the Current Population Survey and the Survey of Income and Program Participation are linked to administrative records on social security earnings and benefits generated by the Social Security Administration.

We use our proposed strategy for handling missing data on pupil's subjective engagement at school at age 16 in the Longitudinal Survey of Young People in England (LSYPE), which have been linked to school administrative records from the National Pupil Database (NPD). Engagement at age 16, which corresponds to the third wave of the survey, is missing for nearly 30% of the sample due to a combination of attrition and item non-response. We expect youngsters to be more likely to drop out and less likely to answer questions regarding their engagement at school when they feel less engaged. As a consequence, there are potential problems of sample selection bias if only available sample data are analysed. We use NPD data on achievement, measured by test scores, as auxiliary information. Achievement is correlated with engagement and is never missing because it comes from the NPD. We investigate the role of income, English language and ethnicity on pupil's subjective engagement at school at age 16. We do not want to control for achievement. Instead we use a joint modeling approach to make the MAR assumption more plausible.

The paper is organised as follows. Section 2 introduces notation, describes our proposed modelling approach and two alternative methods for handling missing data, namely the Heckman selection model and inverse probability of selection weighting. It also presents the main contribution of this paper, which is establishing conditions under which the proposed approach will reduce sample selection bias compared with ordinary least squares. In Section 3, we use our method to model pupil's subjective engagement at school. Next, Section 4 compares our approach with the two alternatives using a Monte Carlo study. Finally, a discussion is given in Section 5.

2. Proposed seemingly-unrelated regressions (SUR) strategy

2.1. General idea

The main objective is to estimate a linear regression model with response variable y_i for individual i ($i = 1, \ldots, N$) and with K covariates $\mathbf{x}_i$ (including the constant). Variable y_i is observed only if a selection condition ($s_i = 1$) is met and is missing otherwise ($s_i = 0$). The condition for the survey response to be missing at random (MAR) can then be written as $\mathrm{P}(s_i \mid y_i, \mathbf{x}_i) = \mathrm{P}(s_i \mid \mathbf{x}_i)$.

When MAR is violated, ignoring the missingness process can lead to inconsistent estimators. In this paper, we suggest making the MAR assumption more plausible by modelling y_i jointly with an auxiliary variable a_i, that is correlated with y_i given $\mathbf{x}_i$ and never (or rarely) missing. The MAR condition then becomes $\mathrm{P}(s_i \mid y_i, \mathbf{x}_i, a_i) = P(s_i \mid \mathbf{x}_i, a_i)$. Let s_i^* be a latent continuous variable such that $s_i = 1$ if $s_i^* > 0$ and $s_i = 0$ otherwise. Under multivariate normality of $(y_i, s_i^*, a_i)'$ given $\mathbf{x}_i$, the conditions for MAR with and without auxiliary information become

$$\mathrm{Cor}(y_i, s_i^* \mid \mathbf{x}_i, a_i) = 0 \tag{1}$$

and

$$\mathrm{Cor}(y_i, s_i^* \mid \mathbf{x}_i) = 0, \tag{2}$$

respectively.

2.2. Selection bias under SUR and OLS

The SUR model consists of several linear regression equations that are linked by allowing their error terms to be correlated. Each equation has its own continuous response and potentially different sets of covariates. We specify two regression equations

$$\begin{aligned} y_i &= \mathbf{x}_i'\boldsymbol{\beta}_y + \epsilon_{yi} \\ a_i &= \mathbf{x}_i'\boldsymbol{\beta}_a + \epsilon_{ai} \end{aligned} \tag{3}$$

where $\boldsymbol{\beta}_y$ and $\boldsymbol{\beta}_a$ are regression coefficients and the error terms ϵ_{yi} and ϵ_{ai} are assumed to have a bivariate normal distribution with zero means, standard deviations σ_y and σ_a, and correlation ρ. We let both regression equations contain the same set of covariates.

We now consider different models for the probability that the response variable y_i is observed. A probit model for s_i can be written as a linear model for the corresponding latent continuous response s_i^*,

$$s_i^* = \mathbf{x}_i'\boldsymbol{\beta}_s + \theta y_i + \alpha a_i + \epsilon_{si}, \tag{4}$$

where $\boldsymbol{\beta}_s$, θ, and α are regression coefficients, the error term $\epsilon_{si} \sim N(0,1)$ is independent of the error terms in (3), and the selection indicator takes the value 1 if $s_i^* > 0$ and the value 0 otherwise.

The model allows selection to depend on the covariates in the model for y_i, on the response variable itself, and on the auxiliary variable a_i. When the regression model for y_i is estimated by ordinary least squares (OLS), data are MAR if $\theta = 0$ and $\alpha = 0$. For the SUR model, MAR requires only that $\theta = 0$.

A necessary condition for the SUR strategy to deliver (sample selection) bias reduction compared with OLS is that the auxiliary response a_i should carry information about the missing response y_i over and above what is already explained by the covariates. In other words, we require

$$\text{Cor}(a_i, y_i \mid \mathbf{x}_i) = \text{Cor}(\epsilon_{ai}, \epsilon_{yi} \mid \mathbf{x}_i) \equiv \rho \neq 0. \tag{C1}$$

Condition (C1), however, does not guarantee bias reduction. Bias results from violation of the mean-independence of the error term ϵ_{yi} for y_i in the selected sample, $E(\epsilon_{yi}|\mathbf{x}_i, s_i = 1) \neq 0$. It can be shown that this expectation, referred to here as 'bias', for OLS and SUR is given by

$$\text{bias}^{\text{OLS}}(\theta,\alpha,\rho) = \left[\frac{\theta\sigma_y^2 + \alpha\rho\sigma_y\sigma_a}{\sqrt{\theta^2\sigma_y^2 + \alpha^2\sigma_a^2 + 2\theta\alpha\rho\sigma_y\sigma_a + 1}}\right] \lambda_i^{\text{OLS}}(\theta,\alpha,\rho) \tag{5}$$

$$\text{bias}^{\text{SUR}}(\theta,\alpha,\rho) = \left[\frac{\theta\sigma_y^2(1-\rho^2)}{\sqrt{\theta^2\sigma_y^2(1-\rho^2) + 1}}\right] \lambda_i^{\text{SUR}}(\theta,\alpha,\rho) \tag{6}$$

with

$$\lambda_i^{\text{OLS}}(\theta, \alpha, \rho) = \lambda_i \left(\frac{\mathbf{x}_i' \boldsymbol{\beta}_s^*}{\sqrt{\theta^2 \sigma_y^2 + \alpha^2 \sigma_a^2 + 2\theta\alpha\rho\sigma_y\sigma_a + 1}} \right) \tag{7}$$

$$\lambda_i^{\text{SUR}}(\theta, \alpha, \rho) = \lambda_i \left(\frac{\mathbf{x}_i' \boldsymbol{\beta}_s^*}{\sqrt{\theta^2 \sigma_y^2 (1 - \rho^2) + 1}} \right) \tag{8}$$

where $\boldsymbol{\beta}_s^* = (\boldsymbol{\beta}_s + \theta\boldsymbol{\beta}_y + \alpha\boldsymbol{\beta}_a)$ and $\lambda(\cdot) = \phi(\cdot)/\Phi(\cdot)$ represents the inverse Mill's ratio with $\phi(\cdot)$ and $\Phi(\cdot)$ denoting the standard normal density and cumulative density functions, respectively.

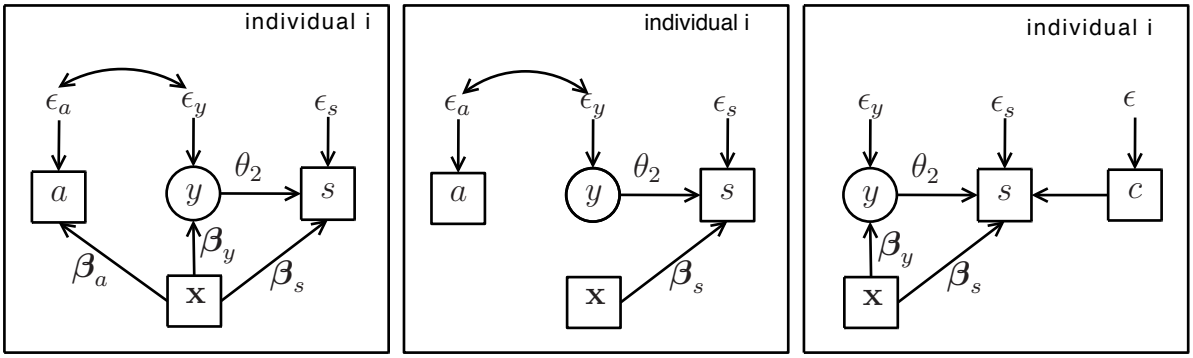
As expected, the bias for the two approaches is zero when the respective MAR conditions hold, i.e., $\text{bias}^{\text{OLS}}(\theta = 0, \alpha = 0, \rho) = 0$ and $\text{bias}^{\text{SUR}}(\theta = 0, \alpha, \rho) = 0$. Also note that when $\theta \neq 0$ and $\alpha = 0$, the bias of OLS is not a function of ρ, whereas the bias of SUR is a function of ρ that attains a maximum when $\rho = 0$, with $\text{bias}^{\text{SUR}}(\theta, \alpha = 0, \rho = 0) = \text{bias}^{\text{OLS}}(\theta, \alpha = 0, \rho = 0)$. In other words, $\text{bias}^{\text{SUR}}(\theta, \alpha = 0, \rho) \leq \text{bias}^{\text{OLS}}(\theta, \alpha = 0, \rho)$. If the data are not MAR for either approach, i.e., $\theta \neq 0$, a sufficient (but not necessary) condition for SUR to be less biased than OLS (which we will refer to as bias reduction) is therefore that $\alpha = 0$. This implies,

$$P(s_i \mid \mathbf{x}_i, y_i, a_i) = P(s_i \mid \mathbf{x}_i, y_i). \tag{C2}$$

The condition is that selection does not depend on a_i once y_i and $\mathbf{x}_i$ are conditioned on. In the setting of equation (4), this holds if $\alpha = 0$. Notice that (C2) is an untestable assumption because, by definition, y_i is missing when $s_i = 0$.

According to Wooldridge (2002b, p. 64) (C1) and (C2) together make a_i an *imperfect proxy* for y_i in the selection equation s_i. A proxy satisfies the additional requirement that $E(y_i \mid a_i, \mathbf{x}_i) = E(y_i \mid a_i)$. Figure 1 (a) shows a path diagram for the SUR model when conditions (C1) and (C2) are met and a_i is an imperfect proxy for y_i, whereas Figure 1 (b) shows the case where a_i is a proxy for y_i. In these diagrams, squares represent observed variables and circles unobserved variables (y_i is in a circle because it is sometimes missing), long arrows represent regressions (linear or probit) and short arrows represent error terms

(in the selection equation, the error does not enter the model for s_i additively). Curved, double-headed arrows connect variables that are correlated.



(a) imperfect proxy variable (b) proxy variable (c) exclusion restriction

Figure 1 Path diagrams. In panel (a) a is an imperfect proxy (conditions C1 and C2) variable for y in the selection equation s; in panel (b) a is a good proxy variable for y in s; and in panel (c) control variable c satisfies the exclusion restriction for two-stage least squares.

It is important to note that conditions (C1) and (C2) are different from requiring an exclusion restriction, which a two-stage least squares approach for handling sample selection would demand (see also Section 2.3). Indeed, an exclusion restriction requires nominating a variable c_i that affects the probability of selection but not the response variable, given the covariates, as depicted in panel (c) of Figure 1. No such exclusion restriction is needed for the SUR strategy to work because the selection mechanism is not explicitly modelled. Instead, a sufficient condition for bias reduction is that ϵ_{ai} is correlated with ϵ_{yi} and that, conditional on y_i and the covariates $\mathbf{x}_i$, selection does not depend on a_i. This assumption is easier to satisfy. Think, for instance, of the classical problem of item non-response to income questions. In general, people are reluctant to give information about their earnings. Moreover, research in the area has consistently found that low and high earners answer income questions at significantly lower rates than people in the middle of the income distribution (see, for instance, Lillard et al. 1986). Fitting an income equation in this context is complicated by the fact that the selection mechanism is a function of the response variable. Finding a variable that affects the selection probability but not income itself, given other controls, is a difficult task.

It is easier, however, to think of an additional response variable that is: (1) correlated with income, (2) unlikely to suffer from item non-response, (3) does not belong to the main equation, and (4) is unlikely to affect selection given income. Say, expenditure on clothing. Hence, this will be a good candidate for an auxiliary variable for implementing the SUR strategy. Notice this auxiliary variable does not comply with the requirements for imposing a valid exclusion restriction because: (1) it is an endogenous variable in the s_i^* equation (because it is correlated with the response and y_i is not observed), and (2) it does not belong in the s_i^* equation.

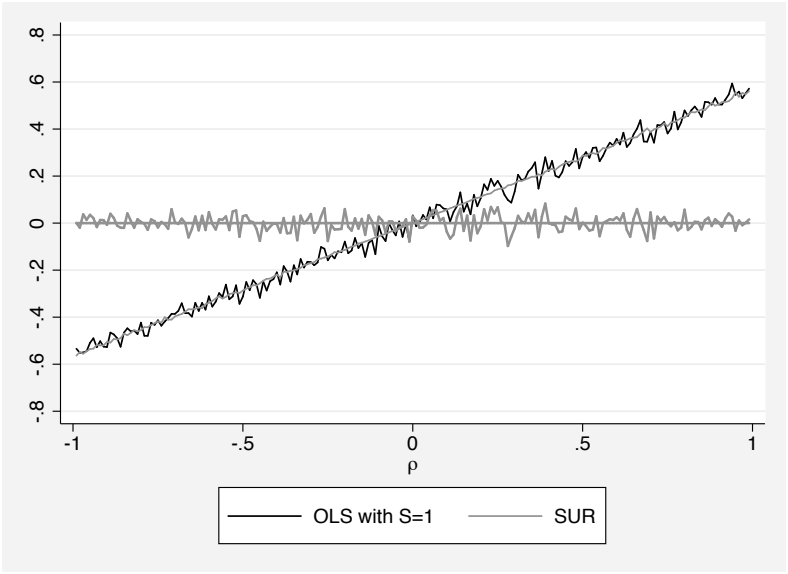
Figure 2 shows the bias for the intercept for OLS (black curves) and SUR (grey curves) as a function of ρ for several different values of θ and α when there are no covariates, the true intercepts are $\beta_{S0} = \beta_{y0} = \beta_{a0} = 0$ and the residual standard deviations are $\sigma_a = \sigma_y = 1$. The smooth curves are the functions given in (5) and (6), whereas the noisy curves are the estimated bias using 1,000 simulated datasets.

The top graph in Figure 2 is for the case where missingness is MAR for the SUR model ($\theta = 0$) but not MAR (NMAR) for OLS ($\alpha = 1$). As expected, the OLS estimator is subject to sample selection bias unless the residual correlation ρ between ϵ_{yi} and ϵ_{ai} is zero. Because y_i is more likely to be observed when a_i takes larger values (with $\alpha > 0$), the bias is positive when ρ is positive and negative when ρ is negative. There is no bias for SUR because the data are MAR for the SUR model.

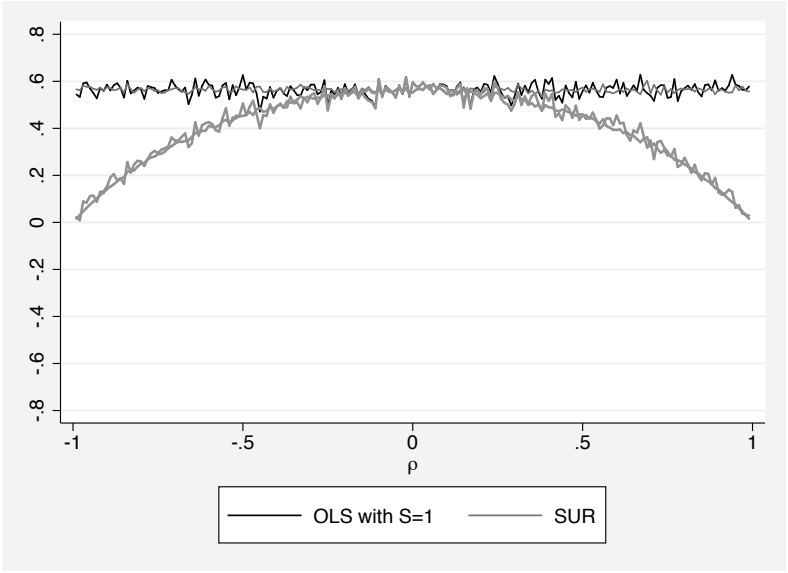
The middle graph in the figure considers the situation where the data are NMAR for the SUR model ($\theta = 1$), but a_i does not affect selection ($\alpha = 0$). Here, OLS is more severely biased than SUR except when $\rho = 0$. Because condition (C2) holds, the SUR model reduces the bias as long as $\rho \neq 0$. In fact, as the correlation between ϵ_{yi} and ϵ_{ai} tends to ± 1, the bias for SUR tends to 0.

In the bottom graph of Figure 2, the data are NMAR for the SUR ($\theta = 1$) and a_i does affect selection ($\alpha = 1$). Figure 2 shows that SUR reduces the bias compared with OLS only when ρ is positive and sufficiently large. Otherwise, OLS outperforms SUR. To understand why this happens, it is useful to look at equation (4) and consider the case that $\text{sign}(\theta) = \text{sign}(\alpha)$ and $\rho < 0$. In such a context, it is clear from equation (4) that a

MAR ($\theta = 0; \alpha = 1$)



NMAR ($\theta = 1; \alpha = 0$)



NMAR ($\theta = 1; \alpha = 1$)

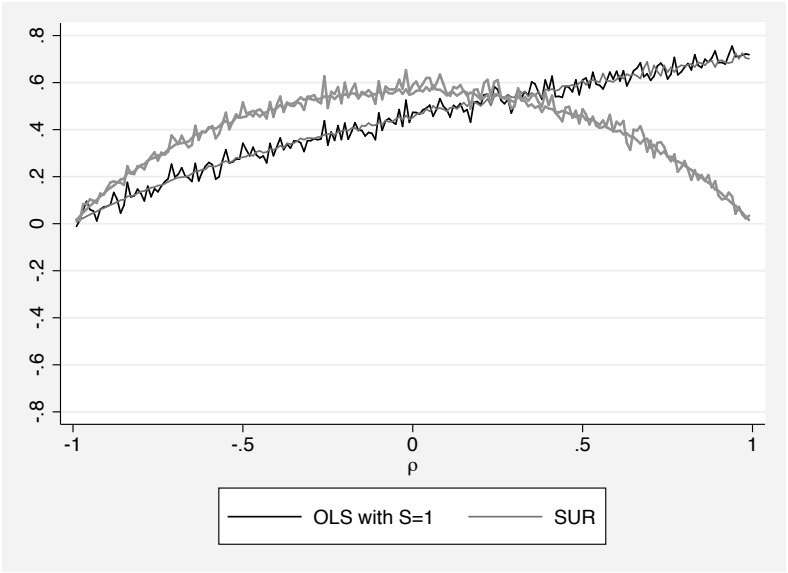


Figure 2 Sample selection bias. Black curves represent OLS bias, grey curves represent SUR bias. Smooth curves represent the theoretical bias and noisy curves represent bias estimates using simulations.

positive (negative) increment of y_i will be partially compensated by a negative (positive) increment of a_i because the two variables are negatively correlated and θ and α have the same sign. This cancelling out makes the MAR assumption more valid. The performance of OLS improves as more of the effect of y_i on s_i is wiped out.[1] Now, because the SUR model has the effect of stripping out the effect of ϵ_{ai} from y_i and s_i, the partial effect of y_i on s_i cannot be counteracted by the partial effect of a_i on s_i. For these reasons, given the right conditions, OLS can outperform SUR when $\alpha \neq 0$.

2.3. Alternative methods for handling missing data

We consider two alternative approaches for handling missing data when auxiliary information a_i is available: inverse probability of selection weighting and the Heckman selection model. Inverse probability of selection weighting involves fitting the probit selection model in (4), with $\theta = 0$, to the whole sample to obtain predicted selection probabilities given covariates and auxiliary information $\widehat{p}_i = P(\mathbf{S}_i \mid \mathbf{x}_i, a_i)$. Then, in a second stage, the equation for y_i in (9) is estimated for the selected sample (with $s_i = 1$) by weighted least squares (WLS) with weights given by $1/\widehat{p}_i$. This will deliver consistent estimators for $\boldsymbol{\beta}_y$ (Horvitz and Thompson 1952)(Little and Robin 1987, p. 57) (Robins and Rotnitzky 1995, p. 123) (Wooldridge 2002a, p. 122). However, because information has been discarded in the second stage, WLS estimators are inefficient. Small $\widehat{p}_i$ and hence large weights can lead to large standard errors (Basu 1971).

The Heckman selection model (Heckman 1979) is a joint model for y_i and s_i^*. If $\theta \neq 0$ and $\alpha \neq 0$, the data are not missing at random, i.e. selection is informative. In the equations for y_i and s_i^*,

$$y_i = \mathbf{x}_i'\boldsymbol{\beta}_y + \epsilon_{yi} \tag{9}$$

$$s_i^* = \mathbf{x}_i'\boldsymbol{\beta}_s^* + \xi_{si} \tag{10}$$

[1] A similar argument shows that if $\text{sign}(\theta) \neq \text{sign}(\alpha)$ and $\rho > 0$ the OLS estimator will benefit from the cancelling out of the effect of y_i on s_i.

the errors ϵ_{yi} and ξ_{si} are correlated because

$$\xi_{si} = \theta\epsilon_{yi} + \alpha\epsilon_{ai} + \epsilon_{si}. \tag{11}$$

Because of the correlation between the error terms, equations (9) and (10) should be fitted as a system in order to obtain a consistent estimator of $\boldsymbol{\beta}_y$. One straightforward option will be estimating the parameters by maximum likelihood (ML).

Notice that when fitting the Heckman model one cannot condition selection on a_i because this variable is correlated with y_i and, as a consequence, is correlated with the composite error $(\theta y_i + \epsilon_{si})$ in the model for s_i^*. In other words, a_i is endogenous in the selection equation and including it as part of the control variables will cause the probit estimator for $\boldsymbol{\beta}_s$ to be inconsistent.

The ML estimator is based on a joint normality assumption for the error terms ϵ_{yi} and ξ_{si} and is efficient. Instead of using ML, it is possible to fit the model by limited information maximum likelihood (LIML), which is a two-step estimation method. In the first step, a probit model is fitted on the whole sample (because s_i and $\mathbf{x}_i$ are always observed) to obtain an estimate for the parameters in the selection equation $\widehat{\boldsymbol{\beta}}_s^*$. In the second step, a least squares estimator is used to fit

$$y_i = \mathbf{x}_i'\boldsymbol{\beta}_y + \delta\lambda_i + u_i \tag{12}$$

using the selected ($s_i = 1$) sample. This regression includes the inverse Mill's ratio,

$$\lambda_i = \frac{\phi(\mathbf{x}_i'\widehat{\boldsymbol{\beta}}_s)}{\Phi(\mathbf{x}_i'\widehat{\boldsymbol{\beta}}_s)},$$

as an additional explanatory variable, or 'control function'. Here, $\delta\lambda_i$ is a correction term that ensures $E(u_i \mid \mathbf{x}_i, \lambda_i) = 0$. As a consequence, once this term is plugged-in, the OLS estimator for $\boldsymbol{\beta}_y$ is consistent. LIML has the advantage of removing the assumption of joint normality for ϵ_{yi} and ξ_{si}. It is enough to suppose that ξ_{si} is normally distributed, and no restrictive distributional assumptions for ϵ_{yi} are needed (Wooldridge 2002b, p.

562) (Vella 1998). Hence, LIML delivers a consistent estimator for $\boldsymbol{\beta}_y$ under weaker conditions than ML. In exchange for the weaker conditions, LIML has the disadvantage of delivering an estimator that is no longer efficient (Vella 1998, Puhani 2000).

Because of the non-linearity of the inverse Mills ratio function, the Heckman model is identified by functional form even if the same set of explanatory variables enter both equations. However, if $\mathbf{x}_i'\boldsymbol{\beta}_s^*$ does not vary sufficiently, λ_i will be well-approximated by a linear function. This can cause severe multicollinearity in (9) and large standard errors for the parameters of interest $\boldsymbol{\beta}_y$ (Wooldridge 2002b, p. 564). As a consequence, the model is likely to give rather unrobust results (Vella 1998, p. 135) (Little 1985, p. 1470) (Puhani 2000, p. 57).[2] In particular, and more substantially, the researcher cannot be confident that evidence for bias, i.e. $\widehat{\delta} \neq 0$, is caused by true sample selection bias rather than model misspecification (Little 1985, p. 1470) (Wooldridge 2002b, p. 564). Specifying at least one exclusion restriction is therefore the ideal approach. This is equivalent to finding an instrument c_i for s_i in the y_i equation, as is graphically depicted in panel (c) of Figure 1. Obviously, finding a valid candidate for imposing the needed exclusion restriction is difficult in most cases.

Even if a valid instrument is available, the ML estimator of (9) and (10) will be very sensitive to misspecification of the distribution of ϵ_{yi} (Little 1985, p. 1473) (Vella 1998, p. 135). Moreover, even though it is more robust, the LIML can also be affected by serious departures from joint normality because the model still assumes that $E(\epsilon_{yi} \mid \epsilon_{si})$ is linear in ϵ_{si} (see, for instance, Wooldridge 2002b, p. 562). If this condition does not hold, LIML will not be consistent. To address this problem, various approaches that relax the linearity of $E(\epsilon_{yi} \mid \epsilon_{si})$ have been suggested. One approach is the semi-parametric two-step selection model discussed by Vella (1998). In the first step, a semi-nonparametric model is fitted using the methods of Gallant and Nychka (1987) to obtain a consistent estimator of the linear index $\mathbf{x}_i'\boldsymbol{\beta}_s^*$ in the binary variable model. Then, as suggested by Newey (2009), the second step fits an OLS regression for y_i in the selected sample, adding powers of $\mathbf{x}_i'\boldsymbol{\beta}_s^*$ to control for selection. This is a control function approach.

[2] Vella (1998) (p. 135) says that in such a case "...the degree of identification is "weak" ..."

3. Application: missing data on pupil's subjective engagement at school

Here we investigate the role of income, English as a second language and ethnicity on pupil's subjective engagement at school at age 16. We use the Longitudinal Survey of Young People in England (LSYPE) linked to the National Pupil Database (NPD). The pupils were aged 16 in the third wave of the LSYPE. A major challenge is that engagement at age 16 is missing for nearly 30% of the sample, due to a combination of dropout and item non-response.

We expect pupils to be more likely to continue participating in the survey and to answer questions regarding their engagement at school when they feel more engaged (or unlikely to respond if they are extremely disengaged). Therefore θ may be positive, leading to sample selection bias. To address the issue, we use achievement scores from administrative data as auxiliary variables. Achievement and engagement are expected to be positively correlated ($\rho > 0$). After controlling for engagement, achievement may not substantially affect the probability of selection, but if it does, we expect the effect to be positive ($\alpha > 0$), so that it is likely that SUR will lead to a bias reduction compared with OLS. We will use achievement scores at ages 14 and 16, denoted a_{1i} and a_{2i}, as auxiliary variables. As another auxiliary variable z_i, we will use engagement at age 14, although this response was missing for 16.4% of the sample. The SUR model for the application is

$$\begin{aligned} y_i &= \mathbf{x}_i'\boldsymbol{\beta}_y + \epsilon_{yi} \\ a_{1i} &= \mathbf{x}_i'\boldsymbol{\beta}_{a1} + \epsilon_{a1i} \\ a_{2i} &= \mathbf{x}_i'\boldsymbol{\beta}_{a2} + \epsilon_{a2i} \\ z_i &= \mathbf{x}_i'\boldsymbol{\beta}_z + \epsilon_{zi} \end{aligned} \tag{13}$$

Where $\boldsymbol{\beta}_y$, $\boldsymbol{\beta}_{a1}$, $\boldsymbol{\beta}_{a2}$ and $\boldsymbol{\beta}_z$ are regression coefficients and the error terms ϵ_{yi}, ϵ_{a1i}, ϵ_{a2i} and ϵ_{zi} are assumed to have a multivariate normal distribution with zero means and an unstructured covariance matrix.

We use the SUR strategy described in Section 2, but with three auxiliary variables: achievement a_{1i} and a_{2i} at ages 14 and 16 and engagement z_i at age 14. Results are then compared with those obtained by alternative estimation methods, including OLS, WLS, the selection model suggested by Heckman (1979), and the semi-parametric two-step selection model of Vella (1998).

3.1. Longitudinal Survey of Young People in England and National Pupil Database

We use data from the LSYPE linked to the NPD. The NPD is the set of administrative records for the whole population of pupils in state maintained schools in England and contains information on a limited number of key variables. The NPD was used as a sampling frame for the LSYPE and the two data sets can be linked at the pupil level.

The target population of the LSYPE is year 9 secondary school students, aged between 13 and 14 in 2004, in state maintained, pupil referral units, and independent schools in England. The LSYPE is a two-stage survey. In the first stage, schools were sampled from strata of deprived / non deprived schools, with deprived schools being over-sampled by a factor of 1.5. In the second stage, pupils from minority ethnic groups (Indian, Pakistani, Bangladeshi, Black African, Black Caribbean, and Mixed) were over-sampled with the objective to achieve 1,000 issued students in each group. The total issued sample had 21,000 students. The design ensures that within stratum and within ethnic group all students have the same probability of selection.

We use data from the LSYPE wave 1 (W1) and wave 3 (W3), i.e. students aged 14 nd 16. Students with serious learning disabilities with a full statement of special education needs (FSEN) are excluded from the analytical sample, but we include pupils with partial special education needs (SEN). A total of 838 schools were sampled to the LSYPE and 647 schools (73%) participated. School non-response was a problem particularly in London where only 57% of the selected schools participated. Out of the 21,000 issued interviews, a total of 15,770 students / households co-operated with the study (74% response rate), yielding 13,914 (66%) full interviews and 1,856 partial interviews (9%). After excluding

1,108 students from private schools and 498 FSEN students we are left with 14,164 observations. This is our analytical sample.

3.2. Variables

Engagement at age 16 is our response variable of interest. Engagement scores, both at age 14 and 16, are constructed from pupil's responses to 8 subjective questions in the LSYPE:

- I am happy when I am at school;
- School is a waste of time;
- School work is worth doing;
- Most of the time I don't want to go to school;
- On the whole I like being at school;
- I work as hard as I can in school;
- In a lesson, I often count the minutes till it ends;
- The work I do in lessons is a waste of time.

Each question has 4 response categories, from 1 (strongly agree) to 4 (strongly disagree). Negative questions were code-reversed. Next, we calculated the (raw) sum of the 8 items. Preliminary inspection showed that the distribution of the raw sum is seriously negatively

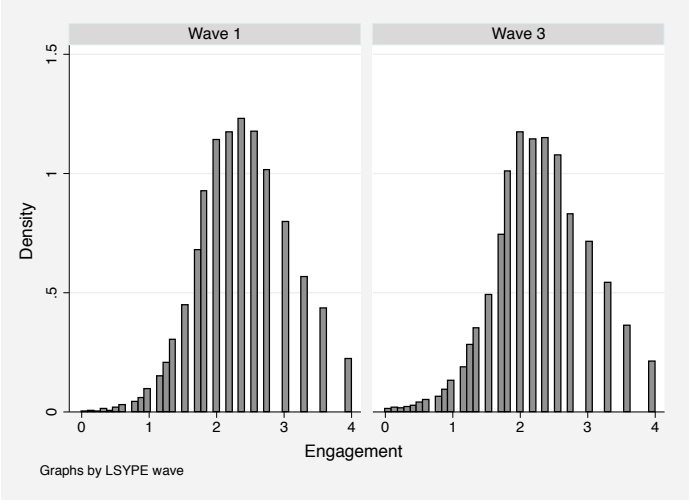


Figure 3. Engagement histogram by LSYPE wave.

skewed. To better comply with an assumption of normality we applied a transformation. In particular, the raw-sum distribution was first reflected and then a square-root transformation applied. Finally, the distribution was reflected again, so that regression signs remain unaffected.[3] Figure 3 shows histograms of the resulting engagement scores at age 14 (denoted z_i) and at age 16 (denoted y_i).

The NPD contains information on examination results at age 14, i.e. Key Stage 3 tests (KS3), and at age 16, i.e. General Certificate of Secondary Education tests (GCSEs). Both KS3 and GCSE tests are externally graded by an independent agency and are a summative measure of pupil's school attainment. We use an average point score (APS) as the auxiliary information brought by the administrative data. To help interpretation, in all our regressions we standardised the KS3 (denoted a_{1i}) and GCSEs scores (denoted a_{2i}). Because achievement information comes from the NPD, a_{1i} and a_{2i} are never missing and we have $14,164$ observations for both 2004 and 2006, corresponding to LSYPE wave 1 and wave 3, respectively.

Explanatory variables come from the NPD to avoid new layers of sample selectivity caused by item non-response in control variables. The NPD contains information on some student characteristics including year and month of birth, gender, ethnicity (14 groups, White British is the reference category), eligibility for free schools meals (a dummy variable that proxies income), English as a second language and special education needs. Previous work on school achievement has found that pupils born in the winter months perform better at exams than children born earlier due to an age within-year effect (see, for instance, Dearden et al. 2011, Crawford et al. 2007, Puhani and Weber 2007). Using knowledge of the month of birth, we define a dummy variable that indicates whether a student was or not born in the winter months (September to December). Nearly 30% of the children in the sample were born in the winter months.

At the school level, we define a dummy variable indicating whether a school is 'deprived', where deprivation is measured as the proportion of pupils receiving free school meals in the school and deprived schools are those in the top quintile of this distribution.

[3]Specifically, we applied the transformation $y_i = 5 - \sqrt{33 - y_i^{\text{raw}}}$ and $z_i = 5 - \sqrt{33 - z_i^{\text{raw}}}$.

Finally, we know the geographic area where the school is located (9 regions, London is the reference category). Again, these variables are never missing and, as a consequence, there are 14,164 observations for both 2004 and 2006.

The British Market Research Bureau (BMRB) was the main contractor for LSYPE wave 1. BMRB subcontracted part of the interview work to other two companies, NOP and IPSOS-MORI. All three companies worked across all geographic areas covered by the LSYPE and in some cases worked together in the field. We have information, for all individuals, on which company approached them for interview. Because contractors have different field experience and incentives to perform, the company that did the interview is likely to be a predictor of item non-response at wave 1. Further, a bad / poor interview experience at wave 1 is probably a factor that determines attrition and item non-response at wave 3. We cannot think why this variable should be associated with a pupil's school engagement or attainment after controlling for the covariates and, as a consequence, it is a good candidate for imposing exclusion restrictions in the Heckman selection model.

3.3. Descriptive results

The response variable y_i, engagement at age 16, is missing for 4,232 (30%) cases, 2,738 (64.70%) due to survey attrition and 1,494 (35.3%) due to survey item non-response. Engagement at age 14, which we use as an auxiliary variable z_i, is missing for 2,021 cases, 15.6% of the sample. Interestingly, the missingness patterns are not monotone. For some pupils z_i is missing, but y_i is not missing (1,261 individuals or 8.9% of the sample). In total, there are 8,671 students for whom we observe engagement at both times.

Table 1 shows the pairwise correlations between engagement and achievement. As expected, achievement at ages 14 and 16 is highly positively correlated, with a correlation coefficient of 0.75. Engagement scores at age 14 and 16 are also positively correlated, though the correlation coefficient is just 0.53. The cross-correlations are all positive as well, ranging between 0.18 and 0.37. Interestingly, the correlation between achievement and engagement is higher at age 16 than at age 14 (0.37 versus 0.18). The reason for this

may be that the GCSE exams represent higher stakes than the KS3 exams.

Table 1 Pairwise correlation coefficients. Number of observations used in the calculation of each correlation coefficient is written in brackets.

	z	y	a_1	a_2
z	1 (11,953)			
y	0.53 (8,671)	1 (9,932)		
a_1	0.18 (11,953)	0.20 (9,932)	1 (14,164)	
a_2	0.27 (11,953)	0.37 (9,932)	0.75 (14,164)	1 (14,164)

Table 2 Mean engagement $\bar{z}$ in W1 and mean (standardised) achievement $\bar{a}_1$ and $\bar{a}_2$ in W1 and W3 by y_i missingness condition. Standard deviations written in parentheses, number of observations written in square brackets, and standard errors written in curly brackets. ‡denotes that the difference in the first two rows is statistically different from zero at the 1% level using a t-test and allowing for unequal variances in the y_i missing and y_i not missing sub-samples.

	$\bar{z}$	$\bar{a}_1$	$\bar{a}_2$
y_i missing	2.23 (0.69)[3,282]	-0.18 (1.00)[4,232]	-0.24 (1.05)[4,232]
y_i not missing	2.34 (0.65)[8,671]	0.15 (0.92)[9,932]	0.19 (0.87)[9,932]
Difference	-0.11‡ {0.01}	-0.33‡ {0.02}	-0.43‡ {0.02}
N. obs	11,953	14,164	14,164

Table 2 presents summary statistics by missingness status of engagement at age 16, y_i. Not missing y_i is associated with better mean achievement at both age 14 and 16 and with a marginally higher mean engagement score at age 14. A t-test at the 1% level rejects the null hypothesis that the difference in means for z_i across the $s_i = 0$ (y_i missing) and $s_i = 1$ (y_i not missing) sub-samples is zero. Similar conclusions are drawn for the achievement variables, a_{1i} and a_{2i}. Descriptive statistics of all variable used in the regressions are given in Table 3.

3.4. Results from regressions

Table 4 reports results from a probit regression for the selection indicator s_i, where y_i is observed only when $s_i = 1$. Among other explanatory variables, we control for previous (age 14) engagement, z_i, previous and current achievement, a_{1i}, a_{2i}, and the company that carried out the LSYPE interview. Previous engagement is significant at the 1% level. As expected, a child who is highly engaged at age 14 has better odds of being

Table 3 Descriptive statistics. Reference category for categorical variables is indicated in brackets

Variable	Obs	Mean	Std.Dev.	Min	Max
		Student characteristics			
z	11953	2.31	0.67	0.0	4.0
y	9932	2.22	0.70	0.0	4.0
a_1	14164	0.05	0.96	-4.0	1.7
a_2	14164	0.06	0.95	-2.8	2.0
s	14164	0.70	0.5	0	1
female	14164	0.50	0.5	0	1
Special Educational Needs	14164	0.11	0.3	0	1
Winter born	14164	0.32	0.5	0	1
Mover year 10	14164	0.02	0.1	0	1
English Additional Language	14164	0.22	0.4	0	1
Free School Meals	14164	0.17	0.4	0	1
White British (reference)	14164	0.62	0.5	0	1
White other	14164	0.02	0.1	0	1
Mixed	14164	0.06	0.2	0	1
Indian	14164	0.07	0.2	0	1
Pakistani	14164	0.07	0.3	0	1
Bangladeshi	14164	0.05	0.2	0	1
Other Asian	14164	0.01	0.1	0	1
Black Caribbean	14164	0.04	0.2	0	1
Black African	14164	0.04	0.2	0	1
Black other	14164	0.00	0.1	0	1
Chinese	14164	0.00	0.0	0	1
Other	14164	0.01	0.1	0	1
Refused	14164	0.01	0.1	0	1
No data	14164	0.01	0.1	0	1
		School and geographic characteristics			
Deprived school	12800	0.09	0.3	0	1
London (reference)	14164	0.18	0.4	0	1
East Midlands	14164	0.09	0.3	0	1
East of England	14164	0.10	0.3	0	1
North East	14164	0.05	0.2	0	1
North West	14164	0.14	0.3	0	1
South East	14164	0.13	0.3	0	1
South West	14164	0.08	0.3	0	1
West Midlands	14164	0.12	0.3	0	1
York	14164	0.11	0.3	0	1
		Company which did the LSYPE interview			
BMRB	14164	0.44	0.5	0	1
NOP	14164	0.44	0.5	0	1
MORI	14164	0.11	0.3	0	1
Other	14164	0.01	0.1	0	1

highly engaged at age 16 and, at the same time, she is less likely to drop out of the LSYPE or not to answer the school engagement questions. Achievement at age 16 is also

a good predictor of the probability of selection. Interestingly, conditional on achievement at age 16, selection does not appear to depend upon achievement at age 14. We find also that children who move schools in year 10, i.e. a year before taking GCSEs, are less likely to be observed. This is intuitive as moving schools may indicate a major disruption for the child as well as the family, leading to an increased likelihood of dropout and item non-response. Among all ethnic groups, only pupils from Black Caribbean and Black African background are significantly less likely to be observed at the 1% level. Students who were interviewed by IPSOS-MORI are more likely to be selected into the sample at a 5% level of significance. All other LSYPE-company dummy variables are statistically insignificant. Importantly, a Wald test for the exclusion of all the LSYPE-company dummy variables gives $\chi^2(3) = 6.5$ (p-value $= 0.09$). So, these dummy variables are only marginally significant in the probit model.

Because, by definition, we do not observe y_i when $s_i = 0$, it is impossible to know whether selection depends on engagement at age 16 after controlling for engagement at age 14 and achievement at ages 14 and 16. Hence, there is no way of testing for a MAR versus NMAR missing data mechanism (or non-informative versus informative selection).

We can, for instance, suppose that the data are NMAR and fit a Heckman sample selection model. Table 5 reports results from such regressions. We do not condition on z_i, a_{1i} and a_{2i} because these variables are endogenous in the model for s_i^* and including them will cause the Heckman model to deliver an inconsistent estimator. The LSYPE-company dummy variables are excluded from the y_i equation and, therefore, are used to identify the model beyond functional form. A Wald test for the exclusion of these dummy variables in the selection equation gives $\chi^2(3) = 9.7$ (p-value $= 0.02$). This suggests that the exclusion restriction may be valid, though the chi-square is relatively low. Keeping these qualifying facts in mind, we conclude from Table 5 that the Heckman selection model gives evidence that there are important sample selection problems caused by attrition and item non-response in these data, with $\text{Cor}(\epsilon_{yi}, \xi_{si})$ estimated as 0.685. Table 5 also reports estimates from the semi-parametric two-step regression suggested by

Table 4 Probit estimates for probability of non-missing engagement variable in LSYPE W3. ‡(†) Significant at 1% (5%). Engagement in W1 is denoted z, achievement in W1 is denoted a_1, and achievement in W3 is denoted a_2.

	Coeff.	SE
Student characteristics		
z	0.078‡	0.008
a_1	-0.035	0.021
a_2	0.291‡	0.021
Female	0.001	0.025
Special Educational Needs	0.083	0.043
Winter born	-0.004	0.027
Mover year 10	-0.304‡	0.095
English Additional Language	-0.088	0.059
Free SchoolMeals	-0.044	0.036
White other	0.052	0.110
Mixed	-0.071	0.056
Indian	0.005	0.071
Pakistani	-0.131	0.074
Bangladeshi	0.084	0.083
Other Asian	-0.052	0.166
Black Caribbean	-0.236‡	0.066
Black African	-0.302‡	0.074
Black other	-0.325	0.203
Chinese	-0.524†	0.238
Other	-0.252	0.154
Refused	-0.192	0.138
No data	-0.023	0.116
School and Geographic characteristics		
Deprived school	-0.014	0.038
Geographic dummy variables	Yes	
Company which did the LSYPE interview		
NOP	0.039	0.027
MORI	0.099†	0.045
Other	-0.141	0.179
N	11,953	

Vella (1998).[4] In line with the Heckman model, the semi-parametric estimator suggests that there is some degree of sample selectivity.

Table 6 reports regression results for OLS regression, WLS regression and SUR. The WLS strategy makes the assumption that the data are MAR once z_i, a_{1i} and a_{2i} have been

[4]Preliminary regressions were fitted using quadratic, cubic, and square power functions of the index function from the first step and the results indicated that the quartic and cubic terms were insignificant. To save space, we report only the regression with a square power function of the first step index.

Table 5 Heckman sample selection and semi-parametric two-step selection estimates for engagement in LSYPE W3 (denoted y_i). Standard errors (SE) are reported. $^{\ddagger}(^{\dagger})$ Significant at 1% (5%). Selection equation contains the same controls as the main equation, plus dummy variables for company which did the LSYPE W1 interview. Wald tests for the exclusion of company which did the LSYPE interview in the selection equation give a $\chi^2(3) = 9.7$ (p-val = 0.02) for the Heckman model and a $\chi^2(3) = 8.4$ (p-val = 0.04) for the semi-parametric model, respectively.

	Heckman		Semi-parametric	
	Coeff.	SE	Coeff.	SE
Student characteristics				
Female	0.081‡	0.015	0.064‡	0.016
Special Educational Needs	-0.288‡	0.025	-0.242‡	0.029
Winterborn	0.003	0.016	0.005	0.015
Mover year 10	-0.444‡	0.065	-0.301‡	0.093
English Additional Language	0.102‡	0.037	0.113‡	0.036
Free School Meals	-0.134‡	0.023	-0.082†	0.033
White other	0.058	0.063	0.059	0.058
Mixed	-0.026	0.034	-0.021	0.031
Indian	0.271‡	0.043	0.224‡	0.047
Pakistani	0.187‡	0.046	0.199‡	0.043
Bangladeshi	0.282‡	0.051	0.201‡	0.073
Other Asian	0.106	0.100	0.081	0.093
Black Caribbean	-0.031	0.043	0.023	0.047
Black African	0.235‡	0.049	0.274‡	0.053
Black other	-0.115	0.141	-0.038	0.134
Chinese	0.106	0.169	0.205	0.161
Other	0.181	0.102	0.230†	0.098
Refused	-0.150	0.088	-0.087	0.085
No data	-0.070	0.065	-0.059	0.060
School and Geographic characteristics				
Deprived school	-0.010	0.024	0.011	0.024
Geographic dummy variables	Yes		Yes	
ρ	0.685‡	0.030		
$\widehat{\mathbf{x}_i'\boldsymbol{\beta}_S}$			0.000	0.038
$(\widehat{\mathbf{x}_i'\boldsymbol{\beta}_S})^2$			0.026†	0.012
N	14,164		14,164	

controlled for. That is, WLS is fitted under the assumption that $\text{Cor}(s_i^*, y_i \mid z_i, a_{1j}, a_{2i}) = 0$ holds. As a consequence, and unlike the Heckman regression, the probit regression in the first step of the WLS estimator conditions on all z_i, a_{1i} and a_{2i}. We refer to SUR for y_i and a_{1i}, a_{2i} and z_i in (13) as "SUR-full". We also fit a SUR model using z_i as the only auxiliary variable, called SUR-z. This approach can be used in longitudinal surveys when

administrative data are not available. SUR-z and SUR-full should work well under both MAR and NMAR if the auxiliary information does not affect the selection mechanism given y_i.

Table 6 Estimates for engagement (y) in LSYPE W3. $^{\ddagger}(^{\dagger})$ Significant at 1% (5%). Engagement in W1 is denoted z, achievement in W1 is denoted a_1, and achievement in W3 is denoted a_2. Control variables from NPD only.

	Linear models				SUR linear models			
	OLS		WLS		SUR-z		SUR-full	
	Coeff.	SE	Coeff.	SE	Coeff.	SE	Coeff.	SE
Student characteristics								
Female	0.070‡	0.014	0.069‡	0.015	0.072‡	0.013	0.071‡	0.013
Special Ed. Needs	-0.245‡	0.023	-0.236‡	0.028	-0.239‡	0.023	-0.241‡	0.022
Winter born	0.005	0.015	0.002	0.016	0.002	0.014	-0.002	0.014
Mover year 10	-0.272‡	0.061	-0.293‡	0.076	-0.257‡	0.058	-0.259‡	0.057
English Add Language	0.114‡	0.034	0.156‡	0.038	0.119‡	0.033	0.124‡	0.033
Free School Meals	-0.087‡	0.021	-0.106‡	0.024	-0.082‡	0.020	-0.089‡	0.020
White other	0.059	0.058	0.061	0.064	0.045	0.057	0.040	0.056
Mixed	-0.021	0.031	-0.009	0.034	-0.020	0.030	-0.018	0.030
Indian	0.242‡	0.039	0.218‡	0.042	0.254‡	0.038	0.262‡	0.038
Pakistani	0.196‡	0.043	0.161‡	0.047	0.195‡	0.041	0.206‡	0.041
Bangladeshi	0.238‡	0.047	0.210‡	0.053	0.225‡	0.046	0.246‡	0.045
Other Asian	0.085	0.092	0.047	0.081	0.091	0.090	0.115	0.089
Black Caribbean	0.030	0.040	0.039	0.041	0.046	0.038	0.040	0.038
Black African	0.294‡	0.045	0.273‡	0.050	0.292‡	0.043	0.317‡	0.043
Black other	-0.030	0.132	-0.099	0.212	-0.031	0.126	0.010	0.125
Chinese	0.208	0.159	0.141	0.155	0.190	0.151	0.161	0.149
Other	0.244†	0.095	0.177	0.119	0.244‡	0.091	0.249‡	0.090
Refused	-0.097	0.082	-0.192†	0.092	-0.068	0.079	-0.078	0.078
Nodata	-0.063	0.060	-0.054	0.071	-0.052	0.058	-0.062	0.058
School and Geographic characteristics								
Deprived school	0.008	0.022	0.005	0.026	0.006	0.021	0.000	0.021
Geographic dummies	Yes		Yes		Yes		Yes	
σ_z					0.652‡	0.004	0.652‡	0.004
σ_y					0.683‡	0.005	0.691‡	0.005
σ_{a_1}							0.872‡	0.005
σ_{a_2}							0.854‡	0.005
$\rho_{z,y}$					0.512‡	0.008	0.513‡	0.008
ρ_{z,a_1}							0.189‡	0.009
ρ_{z,a_2}							0.253‡	0.009
ρ_{y,a_1}							0.215‡	0.009
ρ_{y,a_2}							0.385‡	0.009
ρ_{a_1,a_2}							0.714‡	0.004
N	9,932		8,671		13,214		14,164	
N×T×J					21,885		50,213	

Comparing Tables 5 and 6 we draw the following conclusions. The most affected coefficients are those for special education needs (SEN), mover year 10, free school meals (FSM), and Black African. The correction for sample selection suggested by WLS, SUR, or indeed the semi-parametric two-step selection model for these variables is relatively small and substantially less than that suggested by the Heckman selection model. Consider, for instance, the mover year 10 variable in Table 6. In this case, WLS suggests a negative correction of the OLS estimate of around 0.34 times the standard error (se, hereafter), SUR-z gives a positive correction of 0.25se and SUR-full gives a positive correction of 0.21se. Finally, the semi-parametric two-step selection model suggest a negative correction of 0.47se. The Heckman selection model suggests a negative correction of 2.82se. This is probably the most extreme example across all model specifications. The cases of SEN and FSM are less dramatic. For SEN, WLS suggests a correction of 0.39se whereas SUR-z and SUR-full suggest 0.26se and 0.17se, respectively. The Heckman selection model, in contrast, suggests a correction of -1.87se. For FSM and Black African, we arrive at similar conclusions.

This pattern is repeated across most control variables. Overall, the SUR-full strategy seems to suggest corrections that are of a similar magnitude as WLS and the semi-parametric two-step model. Often, the correction is in the same direction.

4. Monte Carlo simulation study

Here we report a Monte Carlo simulation study that investigates how the SUR strategy performs compared with OLS, WLS, and the Heckman selection model when the missingness process is MAR and NMAR, respectively for the full SUR model. The objective is to learn whether the full SUR model offers a good alternative to the other methods considered in terms of bias, root mean squared error, and coverage of confidence intervals. We also investigate the performance of these estimators under violations of the multivariate normality assumption.

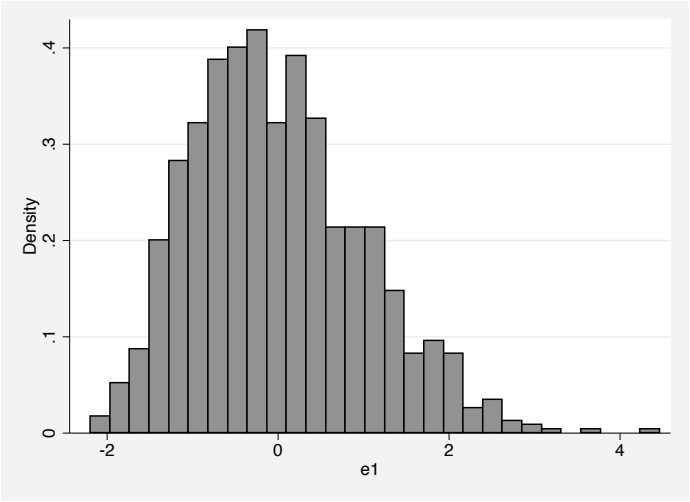


Figure 4 Marginal (zero centred) gamma distribution obtained from a multivariate Frank copula with parameter 1, gamma margins, unit variances, and correlations equal to 0.5. The parameters of the gamma margins are: shape = 9 and rate = 2.

4.1. Approaches compared

We compare the following estimators: (1) ordinary least squares (OLS); (2) weighted least squares with known true weights (WLS-true); (3) weighted least squares (WLS); (4) Heckman sample selection model (Heckman); (5) SUR for z_i and y_i only (SUR-z); (6) SUR for z_i, y_i, a_{1i} and a_{2i} (SUR-full).

4.2. Setup of experiments

Data are simulated from the model specified in (13) under two different assumptions for the error terms $\epsilon_{zi}, \epsilon_{yi}, \epsilon_{a1i}, \epsilon_{a2i}$: (i) multivariate normal distribution, and (ii) multivariate non-normal distribution. In both cases the error terms are generated with zero means, unit variances, and correlations equal to 0.5. To draw from a non-normal multivariate distribution with the required covariance matrix, we follow Mair et al. (2011) and use a Frank copula with parameter 1 and gamma margins. We wish to have clear departures from multivariate normality. As a consequence, the gamma margins are set with shape parameter 9 and rate parameter 2 so that the distribution is positively skewed and has a long right tail. Figure 4 gives a histogram of $1,000$ draws from this distribution, with mean set to 0.

The following aspects of the simulation study remain unchanged regardless of the

distribution of the error terms. Replications are indexed with superscript r, with $r = 1, \ldots, R$, where $R = 1,000$. In each replication four independent standard normal covariates $x^r_{i1}, \ldots, x^r_{i4}$ and three independent Bernoulli(0.5) dummy variables $d^r_{i1}, \ldots, d^r_{i3}$ are simulated, with $i = 1, \ldots, N$ and $N = 1,000$. Next, four multivariate error terms $\epsilon^r_{zi}, \epsilon^r_{yi}$, $\epsilon^r_{a1i}, \epsilon^r_{a2i}$ are generated as described in the previous paragraph. Response and auxiliary variables $z^r_i, y^r_i, a^r_{1i}, a^r_{2i}$ are then obtained using equation (13). Finally, the selection variable $s^r_i = 1(s^{*r}_i > 0)$ is generated on the basis of the simulated responses, an independent standard normal residual ϵ^r_s, and the following equation:

$$s^{*r}_i = \mathbf{x}^{r\prime}\boldsymbol{\beta}_s + \theta_1 z^r_i + \theta_2 y^r_i + \alpha_1 a^r_{1i} + \alpha_2 a^r_{2i} + \epsilon^r_{si}. \tag{14}$$

The selection rule is then used to make y^r_i missing when $s^r_i = 0$.

We vary θ_1, θ_2, α_1 and α_2 to change whether the missingness process is MAR for SUR-full (depending on whether or not $\theta_2 = 0$) and to vary the amount of information that z_i, a_{1i} and a_{2i} provide about selection (reflected by the difference between $\text{Cov}(s^*_i, y_i \mid \mathbf{x}_i)$ and $\text{Cov}(s^*_i, y_i \mid \mathbf{x}_i, z_i, a_{1i}, a_{2i})$). With the residual correlations held constant at 0.5, this depends only on the values of θ_1, α_1 and α_2. Table 7 presents the values of θ_1, θ_2, α_1 and α_2 for each experiment. Whereas experiment 1 and experiment 2 are MAR for SUR-full ($\theta_2 = 0$), experiment 3 to experiment 5 are NMAR for SUR-full and in experiment 3 the auxiliary variables carry the most information about selection. Table 8 reports the values of $\boldsymbol{\beta}$ in each equation and experiment.

Table 7 Parameters θ_1, θ_2, α_1 and α_2 for the Monte Carlo simulations. Experiments 1 and 2 are MAR for the SUR-full ($\theta_2 = 0$), experiments 3 to 5 are NMAR for SUR-full and in experiment 3 the auxiliary variables carry the most information about selection.

					Correlation		
Exp.	θ_1	θ_2	α_1	α_2	$(s^*_i, y_i \mid \mathbf{x}_i)$	$(s^*_i, y_i \mid \mathbf{x}_i, z_i)$	$(s^*_i, y_i \mid \mathbf{x}_i, z_i, a_{1i}, a_{2i})$
1	0.2	0	0.2	0	0.19	0.06	0
2	0.2	0	0.2	0.2	0.27	0.11	0
3	0.2	0.2	0	0	0.28	0.17	0.16
4	0.2	0.2	0.2	0.2	0.42	0.27	0.16
5	0	0.5	0	0.5	0.57	0.47	0.37

Identical covariates $\mathbf{x}^r_i$ are used in all equations. The parameters of substantive in-

terest are $\boldsymbol{\beta}_y$ and these are set to 0.34 across all experiments. The regression coefficients for $\mathbf{x}_i^r$ are positive in all equations and are chosen such that the proportion of variance explained by $\mathbf{x}_i^r$ (coefficient of determination) is on average about 0.41 for z_i^r and y_i^r and about 0.38 for the auxiliary variables a_{1i} and a_{2i}. In the selection equation, $\boldsymbol{\beta}_s$ is set so that the proportion of variance explained by $\mathbf{x}_i^r$, a_{1i}, a_{2i}, z_i^r and y_i^r is about 0.33 and the standard deviation of s_i^{*r} is less than or equal to 1.5, on average.[5] This ensures that an extremely large inverse probability of the selection weight occur rarely so that WLS performs reasonably well (more than 95% of the time $\mathrm{P}(s_i = 1|\mathbf{x}_i)^{-1} \leq 100$ and more than 99% of the time $\mathrm{P}(s_i = 1|\mathbf{x}_i)^{-1} \leq 5003.2$)). The probability of selection is on average 0.7 across all experiments.

Table 8. Values for $\boldsymbol{\beta}_z, \boldsymbol{\beta}_{a1}, \boldsymbol{\beta}_{a2}$ and $\boldsymbol{\beta}_s$ in each Monte Carlo experiment.

				$\boldsymbol{\beta}_s$				
Variable	$\boldsymbol{\beta}_z$	$\boldsymbol{\beta}_{a_1}$	$\boldsymbol{\beta}_{a_2}$	exp 1	exp 2	exp 3	exp 4	exp 5
constant	0.4	0.31	0.25	0.15	0.20	0.15	-0.05	-0.1
x_1	0.4	0.31	0.25	0.10	0.10	0.10	0.10	0
x_2	0.4	0.31	0.25	0.20	0.25	0.20	0.20	0
x_3	0.4	0.31	0.25	0.30	0.30	0.30	0.25	0
x_4	0.4	0.31	0.25	0.40	0.40	0.40	0.35	0
d_1	0.1	0.10	0.47	0.10	0.10	0.10	0.10	0
d_2	0.1	0.31	0.47	0.20	0.20	0.20	0.20	0
d_3	0.1	0.31	0.47	0.30	0.30	0.30	0.30	0

4.3. Results

Figure 5 presents results for one continuous and one dummy variable, i.e. x_1 and d_1, and under the multivariate normality assumption of the error terms. Results for other variables are similar and not presented.[6] We compare estimators in terms of percentage bias, root mean squared error (RMSE) and coverage of the 95% confidence intervals. Note that coverage differs significantly from 95% at the 5% level if it is less than 93.6% or greater than 96.3%.

[5] Notice that $\{\theta_1, \theta_2, \alpha_1, \alpha_2\}$ are fixed by the experiment, so $\boldsymbol{\beta}$ is adjusted to obtain the required variance.

[6] Additional simulation results are available from the authors upon request.

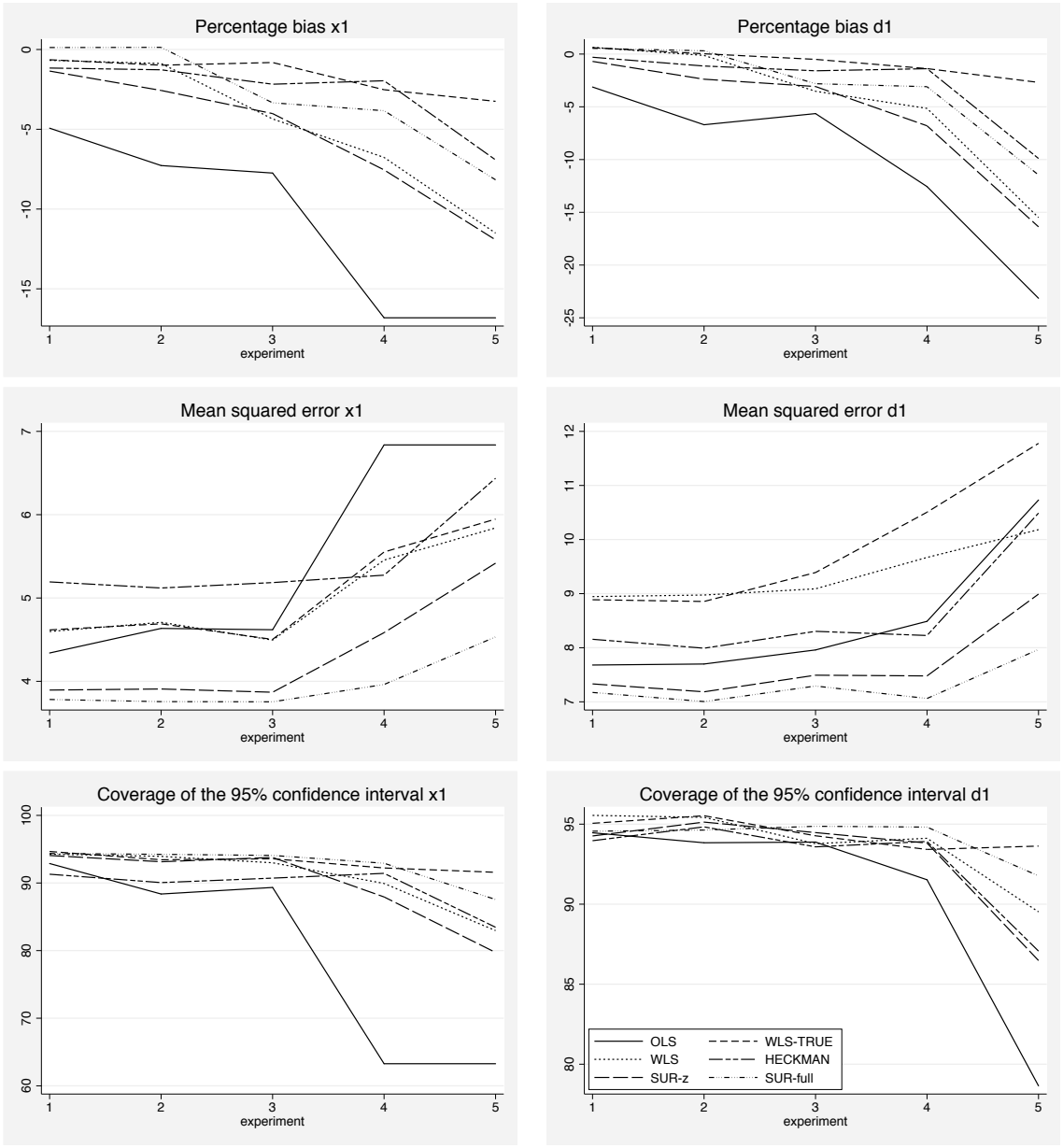


Figure 5 Monte Carlo Simulation Study — multivariate Normal errors. Errors $\epsilon_{zi}, \epsilon_{yi}, \epsilon_{a1i}, \epsilon_{a2i}$ are generated with zero means, unit variances, and correlations equal to 0.5. All other aspects of the simulation study are described in Section 4.2 and in Table 7.

In experiments 1 and 2, y_i is MAR for SUR-full. Figure 5 shows that fitting the equation for y_i in (13) by OLS on the complete data delivers biased estimators. Specifically, there is downward bias for the regression coefficients because individuals with large residuals $\epsilon_{a_{1i}}$ and ϵ_{zi} are selected even if they have small values of $\mathbf{x}_i$ (since $\mathbf{x}_i$, $\epsilon_{a_{1i}}$ and ϵ_{zi} all affect selection positively), and these individuals also tend to have large values of ϵ_{yi} due to the positive correlation among the residuals.

Unsurprisingly, both WLS with known true weights and WLS deliver approximately unbiased estimators that are well behaved. For the Heckman sample selection model we obtain some bias that, however small (no more than 3%), is still significant at the 5% level. Coverage for the Heckman model is also below the advertised 95% level in experiments 1 and 2. SUR-z behaves similarly to the Heckman estimator in terms of bias, but has smaller RMSE and better coverage. Finally, SUR-full produces the smallest bias, which is insignificant at the 5% level for all parameters. The RMSE is also the lowest among all estimators considered and coverage is good for all parameters. These results are as expected since the missingness process is MAR for SUR-full.

When the data are NMAR for SUR-full in experiments 3 to 5, the picture is less clear-cut. Certainly, OLS delivers biased estimators with percentage biases between 13% and 23%. The Heckman model gives a less biased estimator, although bias is significant at the 5% level for all parameters and coverage is below 95% for three parameters in experiment 3, two parameters in experiment 4 and all parameters in experiment 5. SUR-full delivers larger biases than the Heckman estimator and the bias is also significant at the 5% level. However, the RMSEs are smaller than for the Heckman model. In conclusion, SUR- full achieves relatively good bias reduction compared with OLS and fares relatively well when compared to the Heckman sample selection model.

In experiment 3, the data are NMAR for SUR-full and the selection rule does not depend on the auxiliary information. In this case, our theoretical results suggest that SUR-full should be less biased than OLS. Figure 5 supports our predictions. Although the Heckman estimator dominates the SUR-full marginally in terms of bias, SUR-full gives small biases that fluctuate around 5% which seems acceptable. As before, SUR-full dominates the Heckman estimator in terms of RMSE and has marginally better coverage.

In experiments 4 and 5, WLS with known true weights also seems to be a good alternative in terms of bias reduction and coverage. If the true weights are unknown, however, WLS is clearly dominated by both Heckman model and SUR-full. Finally, SUR-z achieves some bias reduction compared with OLS, but SUR-full performs much better.

Figure 6 presents the results from the Monte Carlo study with non-normal errors. The results show that, as expected, the Heckman selection model is quite sensitive to violation of the joint normality assumption with substantial positive bias of around 10% for the coefficients of both x_1 and d_1 across all experiments. SUR-full does not suffer as much from violation of the distributional assumptions. When the data are MAR for SUR-full in experiments 1 and 2, SUR-full is practically not subject to bias. In experiment 3,

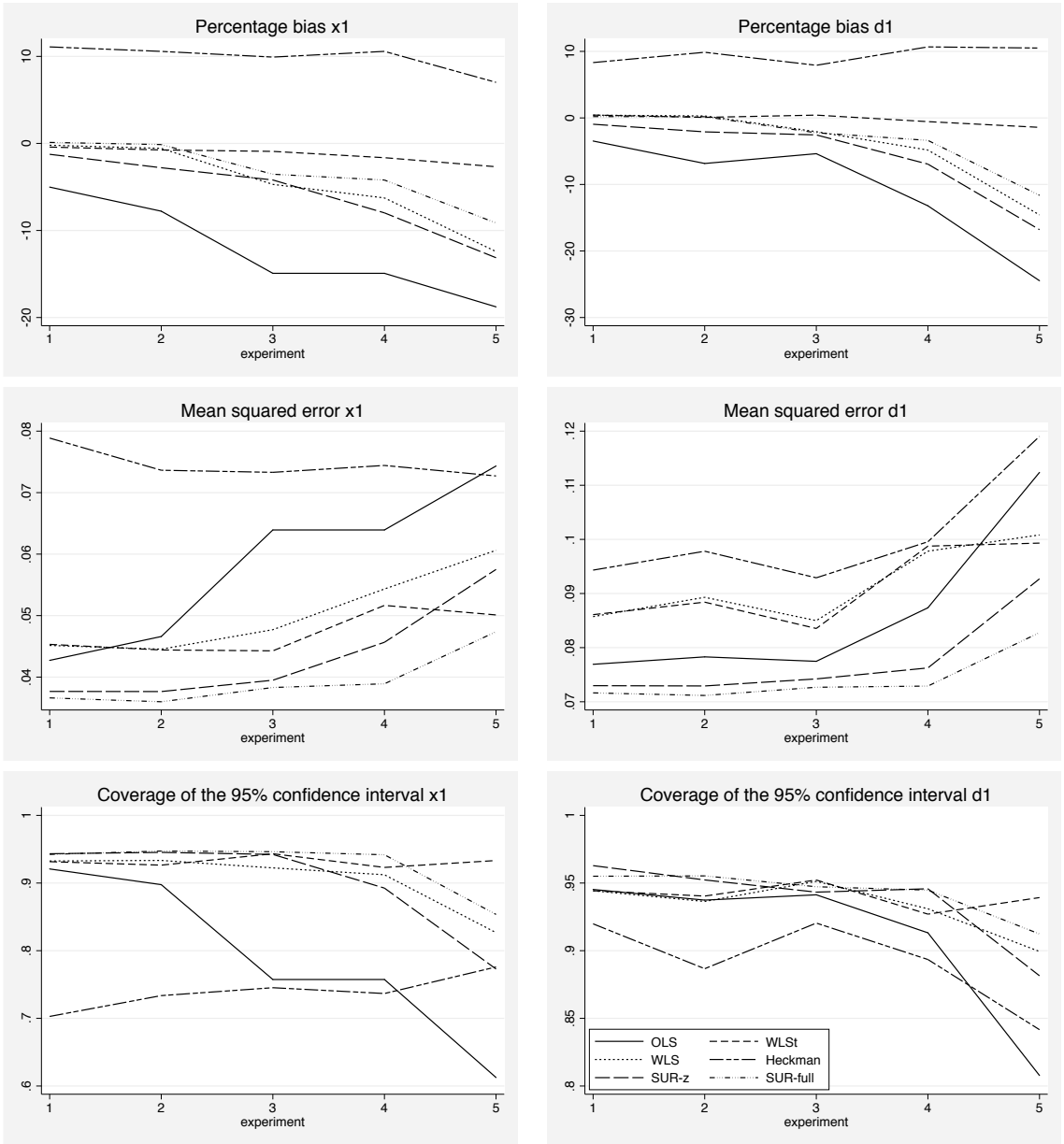


Figure 6 Monte Carlo Simulation Study — multivariate non-Normal errors. Errors $\epsilon_{zi}, \epsilon_{yi}, \epsilon_{a1i}, \epsilon_{a2i}$ Residuals are generated with zero means, unit variances, and correlations equal to 0.5 using a Frank copula with parameter 1 and gamma margins (shape $= 9$, rate $= 2$). The residual in the selection equation 14 is standard normal. All other aspects of the simulation study are described in Section 4.2 and in Table 7.

when the data are NMAR for SUR-full, we still find a moderate bias of -3.53% for the coefficient of x_1 and -2.23% for the coefficient of d_1. It is only in the extreme case of experiment 5 that the SUR-full model has a substantial bias of around 10%. As before, SUR-full is the least biased estimator with the exception of WLS with known weights. Even under violations of the multivariate normality assumption, SUR-full dominates all other estimators as far as the RMSE is concerned and achieves good coverage in all experiments except experiment 5. In contrast, the Heckman model becomes the worst in terms of RMSE and has disappointing coverage. As alternatives to SUR-full, SUR-z and WLS seem to be good options in terms of bias and RMSE.

5. Discussion

The common approaches for dealing with missing data assume that the data are MAR. The MAR assumption is violated if the selection mechanism depends on the response variable after controlling for the covariates. If the data are NMAR, but complete auxiliary information from administrative data linked to the survey data are available, new strategies for handling missing survey data are possible. This present paper proposes fitting a multivariate regression or SUR model for a continuous survey response and continuous auxiliary responses. For the case of one auxiliary variable, we discussed the conditions for this approach to achieve bias reduction compared with OLS.

We used our suggested strategy to deal with problems of attrition and item non-response in the LSYPE, linked to the NPD. In particular, we analysed engagement at age 16 from wave 3 of the LSYPE, using engagement at age 14 from wave 1 of the LSYPE and test scores for ages 14 and 16 from the NPD as auxiliary variables. The bias corrections suggested by our SUR strategy and by inverse probability of selection weighting are relatively small. The SUR bias correction is broadly in line with that suggested by a semi-parametric two-step sample selection model that is robust to deviations from joint normality. In contrast, the Heckman selection model suggests a larger bias correction.

We performed Monte Carlo experiments to compare our approach with the Heckman selection model and with inverse probability of selection weighting. The results sug-

gest that SUR performs well compared with the alternatives considered even when the multivariate normality assumption is violated.

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